OPTIMUM DESIGN FOR STEEL SECTIONS SUBJECTED TO AXIAL COMPRESSIVE LOADS

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ABSTRACT

The design of steel sections subjected to axial compressive loads requires to an initial estimation of the stresses and the dimensions of the sections. Then, the design calculations are performed to check the safety of the estimated section against the applied compressive load, according to Clauses (3-6-4) and (3-7-4) in the Egyptian Steel Specifications, (E.S.S.). Operations of the estimation and check of the sections may be repeated until the safe section is obtained. The amount of safety of the selected section, (i.e., the percentage of value of the permissible compressive stress –to- that of the actual stress of the section), depends on the accuracy of the designer. Thus, an extensive time and more design calculations are carried out in order to obtain the minimum safe area of the section that is called the optimum section, (T.O.S.).

The idea of the research is to convert the non-linearity in the design of compression members to a direct solution for the design of these members. A simple design equation, (D.E.) is formulated to provide, directly, T.O.S. with no need to more calculations. The D.E. depends, mainly, on shape of the section and includes the design criteria of compression members, in addition to the design data.

The D.E. is represented, graphically by charts to determine the best dimensions of common steel shapes of sections used for compression members, such as: single angle, double angles, circular tubes, and **I**- section.

Numerical examples on the design of compression members are solved using the design charts, then, the stresses of the selected sections are checked by the classic manner of the design. It is clearly found that the design of compression members by graphical charts is more easy to use, saves in the design calculations, as well as, it provides T.O.S.

KEYWORDS : Egyptian Steel Specification (E.S.S), the optimum section (T.O.S), design equation (D.E) , the permissible compressive stress , the actual stress, design charts.

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INTRODUCTION

At the beginning of the design of compressive members, shape and dimensions of the section should be estimated in order to evaluate the section properties and, then, the permissible compressive stress is calculated. The actual stress is calculated for that section and it is compared with the value of permissible compressive stress. The described procedure may be repeated if the value of the actual stress is larger than that of the permissible compressive stress of the estimated section. The safe section is obtained when the value of the actual stress is smaller than that of the permissible compressive stress of the section. Thus, the non-linearity of the current design equations described in Clause (3-6-4) in the E.S.S. leads to the following problems:

1. An extensive time and more calculations are carried out during the design of compression members.
2. The amount of safety of the selected section may be varied according to the accuracy of the designer.

In the present research, problems of the current equations of design in the E.S.S. are avoided by converting such non-linear equations to new form of design equations, which provides the direct solution for dimensions of the section. Also, the actual stress of the section is inserted into the equations of permissible stress, so that the minimum safe area of the section is obtained, i.e., T.O.S.

The D.E. is represented graphically by charts for various common shapes of steel sections to provide more simplicity, beside more accuracy for the design of compression members.

FORMULATION OF THE DESIGN EQUATIONS FOR AXIAL COMPRESSIVE MEMBERS

Consider that a compressive load, P, is applied at the center of the cross sectional area, A, of a steel member having buckling length , L_k, about the k-axis of its section. The applied load is perpendicular to the cross section of the member so that the compressive stress is uniform and no shear stress occurs. It is required to design the section of that member where, F_t , is the allowable stress in tension.

The permissible compressive stress, F_c , according to the E.S.S. is defined as the following:

$$F_c = \mu S [F_t - \quad {}_{\kappa}^{2}] \qquad \text{for} (\quad {}_{\kappa} \quad 100) \qquad (1\text{-}a)$$

$$F_c = 7500 \mu S \quad {}_{\kappa}^{2} \qquad \text{for} (\quad {}_{\kappa} \quad 100) \qquad (1\text{-}b)$$

(1)

$$ {}_{\kappa} = L_{\kappa} \,/\, \quad {}_{\kappa} \qquad (2)$$

where,

$_{\kappa}$ is the slenderness ratio of the member about the k-axis,

$_{\kappa}$ is the radius of gyration about the k-axis of the section, and,

is a coefficient depends on the grade of the steel.

$$ = [F_t - 0.75] \,/\, 10^4 \qquad (3)$$

Where, F_t in ton / cm.^{2} , and S is a factor for design defined as:

S = 1 (for the design according to the primary stresses)

S = 1.15 (for the design according to the primary and additional stresses)

μ is the factor of symmetry of the section about the center of the end gusset plate,

μ = 1 (for symmetric sections)

μ = 0.6 (for unsymmetric sections)

On the other hand, the actual compressive stress of the section, F_a,is expressed as:

$$F_a = P \,/\, A \qquad (4)$$

The section is accepted if the value of the actual stress is smaller than that of the permissible compressive stress, i.e.,

$$P \,/\, A \quad \mu \quad S\,[\,F_t - \quad {}^2_{\kappa}\,] \qquad \text{for } (\ \ _{\kappa} \quad 100\,) \qquad (5\text{- a})$$

and, (5)

$$P \,/\, A \qquad 7500\,\mu\, S \qquad {}^2_{\kappa} \qquad \text{for } (\ \ _{\kappa} \quad 100\,) \qquad (5\text{- b})$$

If the area of the section, A, and the radius of gyration, $_{\kappa}$, are put in the forms:

$$A = \quad .\, a\, .\, t \qquad (6\text{-a})$$

(6)

$$_{\kappa} = \quad _{\kappa}\, .\, a \qquad (6\text{-b})$$

where,

a and t are the dimensions of the dependant element in the section, and,

and $_{\kappa}$ are coefficients of the area and the radius of gyration about the k-axis, respectively, and they are evaluated for various common shapes of steel sections with the appropriate corrections. Table (4) indicates values of the coefficients, and $_{\kappa}$ for the area and the radii of gyration for various common shapes of steel sections.

Substituting Eq. (6-b) into Eq. (2), the following relation is obtained.

$$L_{\kappa} \,/\, a = \quad _{\kappa}\, .\quad _{\kappa} \qquad (7)$$

Substituting Eqs. (2) and (6) into Eq. (5) and arranging it so that Eq. (5) is converted to the following formula:

$$(\,L_{\kappa} \,/\, a\,)^2 + (\ \ {}^2_{\kappa} \,/ \quad . \quad)\ \frac{P \,/\, \mu S}{t\,.\,L_{\kappa}}\ (\,L_{\kappa} \,/\, a\,) - \ \ {}^2_{\kappa}\,.\,F_t \,/ \qquad 0 \qquad \text{for } (\ \ _{\kappa} \quad 100\,) \qquad (8\text{-a})$$

$$L_{\kappa} \,/\, a \qquad \left[\frac{7500 \quad {}^2_{\kappa}}{(\,P \,/\, \mu\, S\,) \,/\, t\, L_{\kappa}}\right]^{1/3} \qquad \text{for } (\ \ _{\kappa} \quad 100\,) \qquad (8\text{-b})$$

It is obvious that Eq. (8-a) is a second order equation of L_{κ} / a where the solution of that equation can be determined. Also, by equating Eq. (7) and Eq. (8-b) for $_{\kappa} = 100$, the limiting value between Eq. (8-a) and Eq. (8-b) is expressed by another form. The final form of Eq. (8) becomes as the following:

$$L_\kappa \,/\, a \quad \frac{\overline{[(P/\mu S)/t L_\kappa]^2 + 4 \;\; ^2 \;\; F_t / \;\; ^2{}_\kappa} - [(P/\mu S)/t L_\kappa]}{2 \quad / \; ^2{}_\kappa} \qquad \text{for} \;\; (P/\mu S)/t L_\kappa \quad 7.5\text{x}\, 10^{-3} \;\; / \;\; \kappa \qquad (9\text{-a})$$

$$L_\kappa \,/\, a \quad \left[\frac{7500 \quad ^2{}_\kappa}{(P/\mu S)/t L_\kappa}\right]^{1/3} \qquad \text{for} \;\; (P/\mu S)/t L_\kappa \quad 7.5\text{x}\, 10^{-3} \;\; / \;\; \kappa \qquad (9\text{-b})$$

N.B. [ton and cm. units are used in Eq. (9)].

Eq. (9) represents the D.E. for the axially compressed members, which provides, directly, the dimensions of the safe section. The optimum dimensions of the section are determined by taking the equal symbol in Eq. (9) where the actual stress of the applied load and the permissible compressive stress of the section have the same value.

In order to save the calculations and simplify the design operation of compressive members, the D.E. in Eq.(9) is represented graphically by charts, as in Figs. (1) to (4). The graphical charts are plotted for various shapes of steel sections to provide the optimum values of the ratio, L_κ/a, against different values of the ratio,$(P/\mu S)/t L_\kappa$, for both the principal axes of the section as well as for various grades of steel.

According to the E.S.S., Eq.(9-b) is valid up to $\;\;_\kappa = 180$, and by solving Eqs. (7) and (9-b) for that value of $\;\;_\kappa$, the following condition for the design is obtained.

$$[(P/\mu S)/t L_\kappa]_{mini..} = 1.29\text{x}\, 10^{-3} \;\; / \;\; \kappa \qquad (10)$$

THE OPTIMUM DESIGN OF I- SECTION

I- section is the most common section used for the steel structures. It consists of a single web plate and two equal flange plates which are perpendicular to the x- and y- axes, respectively. The ratio between the web area to the flange area is defined as the following :

$$k = A_w / A_f \qquad (11)$$

The total area of the section, A, is expressed as,

$$A = A_w + 2A_f \qquad (12)$$

In order to design the **I**- section by using Eq. (9), one of the following two ways is considered :

1. The dimensions of the web element are determined by Eq. (9), considering that the member will buckle about the x-axis of the section. Then, dimensions of the flanges are calculated according to Eq. (11).
2. The dimensions of the flange element are determined by Eq. (9), considering

that the member will buckle about the y-axis of the section. Then, dimensions of the web element are obtained using Eq.(11).

1. Web Element Method

Consider an **I-**section with height of web, h_w, and thickness of web, t_w, where they are related by the following ratio:

$$G_w = h_w / t_w \qquad (13\text{-a})$$

According to Clause (3-7-4) in the E.S.S., the maximum value of G_w is defined by the following relation:

$$G_{w_{max.}} \quad 68 / \sqrt{F_y} \qquad (13\text{-b})$$

Substituting Eq. (11) into Eq. (12), the total area of the section and the corresponding coefficient, $_x$, are expressed as,

$$A = [\,(k + 2) / k\,]\ h_w \,.\, t_w \qquad (14)$$

$$_x = (k + 2) / k \qquad (15)$$

Consider that the x-axis is the weak direction for buckling of the member, the moment of inertia of the section, I_x, is obtained, with an appropriate correction, by the following form:

$$I_x = t_w h_w^3 /12 + 2 A_f\ (1.035\, h_w / 2)^2$$

$$I_x = [(k + 6.42) / 12 k]\, A_w .h_w^2 \qquad (16)$$

Dividing Eq. (16) by Eq. (14), and taking root of the result, the radius of gyration, $_x$, and the corresponding coefficient, $_x$, are expressed as the following:

$$_x = 0.288\, h_w \sqrt{(k + 6.42) / (k + 2)} \qquad (17)$$

$$_x = 0.288 \sqrt{(k + 6.42) / (k + 2)} \qquad (18)$$

Introducing the coefficient, G_w, into Eqs. (9-a) and (9-b) and inserting L_x and h_w instead of L_k and a, respectively, the final form of the equation of design of the axially compressed members with **I-**section using web element method is obtained.

$$h_w / L_x \quad \frac{\sqrt{(G_w P / S L_x^2) + {}_x / {}_x^2}}{_x \,.\, F_t} \quad \text{for } \{ G_w P / S L_x^2 \quad 7.5\text{x}10^{-5}\ {}_x / {}_x^2 \} \qquad (19\text{-a})$$

$$h_w / L_x \quad \left(\frac{G_w P / S L_x^2}{7500\ {}_x\ {}_x^2}\right)^{1/4} \quad \text{for } \{ G_w P / S L_x^2 \quad 7.5\text{x}10^{-5}\ {}_x / {}_x^2 \} \qquad (19\text{-b})$$

Eq. (19-b) is valid up to, $_{x}$ = 180, where, the minimum value of the ratio, $G_w\, P/S\, L_x^2$ occurs.

$$[\, G_w\, P/S\, L_x^2\,]_{mini.} = 7.14 \times 10^{-6} \quad {}_{x} / \quad {}_{x}^{2} \qquad (20)$$

It should be noted that Eqs. (19-a) and (19-b) are used for the case of ($_{x}$ $_{y}$), i.e.,

$$L_x / \ {}_{x}\, h_w \qquad L_y / \ {}_{y}\, b_f$$

$$b_f \qquad L_y \qquad \overline{(\, k + 6.42\,) / 2}\ .\ (\, h_w / L_x\,) \qquad (21)$$

where, L_y is the buckling length of the member about the y-axis of the section, and, b_f is the breadth of the flange plates.

Eqs. (19-a) and (19-b) are represented graphically by Fig. (5), which relates the optimum value of, h_w / L_x, and the ratio, $G_w\, P/S\, L_x^2$, for the case of k = 1.

2. Flange Element Method

Consider that, b_f and t_f are breadth and thickness of the flange plates of an **I**-section, where, they are related by the following ratio:

$$G_f = b_f / t_f \qquad (22)$$

Taking the same described procedures for deducing the web element method, it is found that Eqs. (19-a) and (19-b) are valid for the design of **I-**section by flange element method by substituting, b_f, L_y, G_f, $_{y}$, and $_{y}$ instead of h_w, L_x, G_w, $_{x}$, and $_{x}$, respectively, where,

$${}_{y} = \quad k + 2 \qquad (23)$$

$${}_{y} = 0.408 / \quad \overline{k + 2} \qquad (24)$$

Eq. (19-b) is valid up to, $_{y}$ = 180, where, the minimum value of the ratio, $G_f\, P/S\, L_y^2$ occurs.

$$[\, G_f\, P/S\, L_y^2\,]_{mini.} = 4.3 \times 10^{-5} \quad {}_{y}^{2} \qquad (25)$$

It should be noted that $_{y}$ $_{x}$, i.e.,

$$L_y / \ {}_{y}\, b_f \qquad L_x / \ {}_{x}\, h_w$$

$$h_w \qquad L_x \qquad \overline{2 / (\, k + 6.42\,)}\quad (\, b_f / L_y\,) \qquad (26)$$

Figure(6) presents the optimum value of, b_f / L_y, and the ratio, $G_f\, P/S\, L_y^2$, for the case of k = 1.

NUMERICAL EXAMPLES

Example (1)

Miscellaneous examples for the design of compressive members are tabulated in Table (1), where, it indicates values of the applied load, and the effective buckling lengths about the x- and y- axes, respectively. Grade of the used steel as well as the case of stresses included in the design are presented in the same table. It is required to design the optimum section using the graphical method, then, check the safety of that section using the classical manner in the design of compressive members.

Table (1):

Example	P (ton)	Case of Stresses	L_x (cm.)	L_y (cm.)	Steel Grade	Shape of Section
A	10	I	500	700	37	2-angles, back-to-back
B	10	II	L_u = 300	L_v = 150	44	Single angle, (1.5:1)
C	35	II	400	200	37	2-angles, star-shape (2:1)
D	6	I	L_u = 250	L_v = 150	52	Single angle, (1 :1)
E	15	I	600	600	37	Circular tube

Case (I) : ordinary stresses are taken in the design.
Case (II): ordinary and secondary stresses are taken in the design.

The coefficients of the area and radii of gyration are obtained for the required sections, using Table (4-a). The larger value of, L_x / $_x$ and L_y / $_y$, is taken to determine the weak axis for the buckling of the member, subsequently, the corresponding design curve is determined. The thickness of the section is assumed, then, the ratio [(P ⁄ μ S) ⁄ t L_κ] is calculated. The leg length of the angle is obtained from the design curve by knowing [(P ⁄ μ S) ⁄ t L_κ]. The design procedure is tabulated in Table (2), for the presented examples.

Table (2):

Example		$_x$	$_y$	L_x / $_x$	L_y / $_y$	t	P ⁄ μS t L_κ	L_k / a	Section
A	3.76	0.30	0.47	1666.6	1489.4	1.0	0.02	50.2	2-angles 100x100x10
B	2.38	$_u$=0.510	$_v$=0.213	588.2	704.2	0.8	0.12	18.7	Single angle 120x80x8
C	5.76	1.03	0.41	388.3	487.8	0.8	0.19	31.1	2-angles 130x65x8
D	1.88	$_u$=0.385	$_v$=0.195	649.4	769.2	0.8	0.083	18.93	Single angle 80x80x8
E	3.141	0.3536	0.3536	1696.8	1696.8	1.2	0.021	52.1	Circular tube 115x12

In order to check safety of the sections in Table (2), the classical manner of design is used and its steps are presented in Table (3) for the given examples.

Table (3):

Example	Section	P	A	$_x$	$_y$	$L_x/\ _x$	$L_y/\ _y$	F_c	F_a
A	2-angles 100x100x10	10	38.4	3.04	4.57	164.47	153.17	0.277	0.260
B	Single angle 120x80x8	10	15.5	$_u$ = 4.1	$_v$ =1.72	73.2	87.2	0.658	0.645
C	2-angles 130x65x8	35	30.2	6.64	2.61	60.97	76.43	1.171	1.159
D	Single angle 80x80x8	6	12.3	$_u$=3.06	$_v$=1.55	81.7	96.77	0.501	0.488
E	Circular tube 115 x 12	15	43.35	4.07	4.07	147.42	147.42	0.346	0.346

Example (2)

Redesign Example (1-D) considering that the graphical chart for design of single angle with equal legs (1:1) is not available.

Solution

Using the graphical chart for design of single angle with unequal legs (1.5:1),Fig.(1), with some modification for the given values of load and lengths as the following:

P_m = P . [$]_{1.5:1}$ / [$]_{1:1}$ …… P_m = 6 x 2.38/1.88 = 7.596 ton
$L_{u\,m}$ = L_u .[$_u\,]_{1.5:1}$ / [$_u\,]_{1:1}$ …… $L_{u\,m}$ = 250 x 0.51/0.385 = 331.2 cm.
$L_{v\,m}$ = L_v .[$_v\,]_{1.5:1}$ / [$_v\,]_{1:1}$ …… $L_{v\,m}$ = 150 x 0.213/0.195 = 163.8 cm

The same steps in the design of compression members are taken using the modified values of load and lengths.

$L_{u\,m}$ / [$_u\,]_{1.5:1}$ =331.2 / 0.51 = 649.4
$L_{v\,m}$ / [$_v\,]_{1.5:1}$ =163.8 / 0.213 = 769.0 …… use (the v-axis) curve.
Assume thickness of the angle, t = 0.8 cm.
(P/μ S) $/$t $L_{v\,m}$ = (7.596 / 0.6) / (0.8x 163.8) = 0.097
From the v-axis curve in Fig.(1), $L_{v\,m}$ / a = 20.6
The minimum length of the leg, a = 7.95 cm.
Select single angle with equal legs 80 x 80 x 8

Remark:

It is obvious from Example (2) that, the design of compression members using the graphical charts is more flexible so that the optimum dimensions of the required section can be obtained from the graphical chart of another section.

Example (3)

Design an I-section for a column to carry 50 ton. Consider the effective buckling lengths about the x- and y- axes are 12 m. and 6 m., respectively, and steel 37 is used.

1. Design of **I**-section by web element method

Assume, k = 1 …….. $_x$ = 3 and, $_x$ = 0.453

Assume, $G_w = 33$ ……. $G_w P \diagup S L_x^2 = 33 \times 50/ (1200)^2 = 1.14\times10^{-3}$ (S=1)

Using the graphical chart for design of I-section by the web element method, Fig. (5),

$h_w / L_x = 0.022$ ………. $h_w = 26.8$ cm. …….. $t_w = 26.8 / 33 = 0.8$ cm

***************** Select, web plate 270 x 8 *****************

$A_f = A_w / k$ ………. $A_f = 27 \times 0.8 = 21.6$ cm.^{2}

According to Eq. (21), $b_f \quad L_y \quad \overline{(k + 6.42) / 2 .(h_w / L_x)}$

$b_f = 25.4$ cm. , and $t_f = 21.6 / 25.4 = 0.85$ cm.

***************** Select, 2-flange plates 255 x 9 *****************

########## Check of stresses using the classical manner of design ###########

For the selected section, A = 67.5 cm.^{2} , $_x = 12.32$ cm. , $_y = 6.07$ cm.

$L_x / \ _x = 1200 / 12.32 = 97.40$, $L_y / \ _y = 600 / 6.07 = 98.85$ (buckling about y-axis)

$F_c = 1.4 - 6.5 \times 10^{-5} (98.85)^2 = 0.765$ t./ cm.^{2} , $F_a = 50 / 67.5 = 0.74$ t./ cm.^{2}

<u>2. Design of **I**-section by flange element method</u>

Assume, k = 1 …….. $_y = 3$ and, $_y = 0.236$

Assume, $G_f = 22$ ……. $G_f P \diagup S L_y^2 = 22 \times 50/ (600)^2 = 3.06\times10^{-3}$ (S=1)

Using the graphical chart for design of I-section by the flange element method, Fig. (6),

$b_f / L_y = 0.039$ ……… $b_f = 23.7$ cm. …….. $t_f = 23.7 / 22 = 1.08$ cm

***************** Select, 2-flange plates 240 x 11 *****************

$A_w = k . A_f$ ………. $A_w = 24 \times 1.1 = 26.4$ cm.^{2}

According to Eq. ( ), $h_w \quad L_x \quad \overline{2 / (k + 6.42) .(b_f / L_y)}$

$h_w = 24.6$ cm. , and $t_w = 26.4 / 24.6 = 1.07$ cm.

***************** Select, web plate 250 x 11 *****************

########## Check of stresses using the classical manner of design ###########

For the selected section, A = 80.3 cm.^{2} , $_x = 11.40$ cm. , $_y = 5.61$ cm.

$L_x / \ _x = 1200 / 11.40 = 105.3$, $L_y / \ _y = 600 / 5.61 = 107.0$ (buckling about y-axis)

$F_c = 7500 / (107)^2 = 0.655$ t./ cm.^{2} , $F_a = 50 / 80.3 = 0.622$ t./ cm.^{2}

<u>Remark:</u>

It is obvious from Example (3) that, the selection of the ratios, G_w and G_f , depends on the engineering sense and the higher the value of these ratios, the lower the area of the section, and vice-versa.

CONCLUSIONS

The following conclusions can be deduced from the research:

1. Simple equations are deduced to design the axially compressed members according to the Egyptian Steel Specifications, which provide direct solution for the optimum section with high accuracy.

2. The deduced equations are represented graphically, as design charts to determine the optimum dimensions of the section, satisfying the design requirements, for various common steel sections.
3. The new trend in the design of compression members using the graphical charts saves the extensive time and calculations during the design of compression members.
4. The design of compression members using the graphical charts is more flexible so that, the graphical chart of a certain shape of sections can be used for determining the optimum dimensions of another shape of sections with some modifications in values of the load and the lengths.

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APPENDIX

Table (4-a): Coefficients, , and, κ^*, for angle sections.

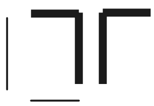
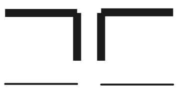
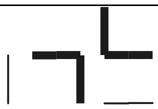
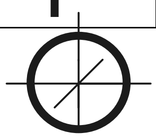
Shape of Section	b/a		x	y
	1.0 1.5 2.0	1.88 2.38 2.88	u 0.385 0.510 0.660	v 0.195 0.213 0.210
	1.0 1.5 2.0	3.76 4.76 5.76	0.300 0.473 0.640	0.470 0.425 0.410
	1.0 1.5 2.0	3.76 4.76 5.76	0.300 0.283 0.263	0.470 0.750 1.030
	1.0 1.5 2.0	3.76 4.76 5.76	0.470 0.750 1.030	0.470 0.425 0.410
	1.0 1.5 2.0	3.76 4.76 5.76	0.470 0.425 0.410	0.470 0.750 1.030
		3.141	0.3536	0.3536

Table (4-b): Coefficients, κ, and, κ, for **I**-section.

	
x	[k + 2] / k
x	0.288 k+6.42 / k + 2
y	k + 2
y	0.408 / k+2

* Coefficient, κ, is taken for the principal axes of section.

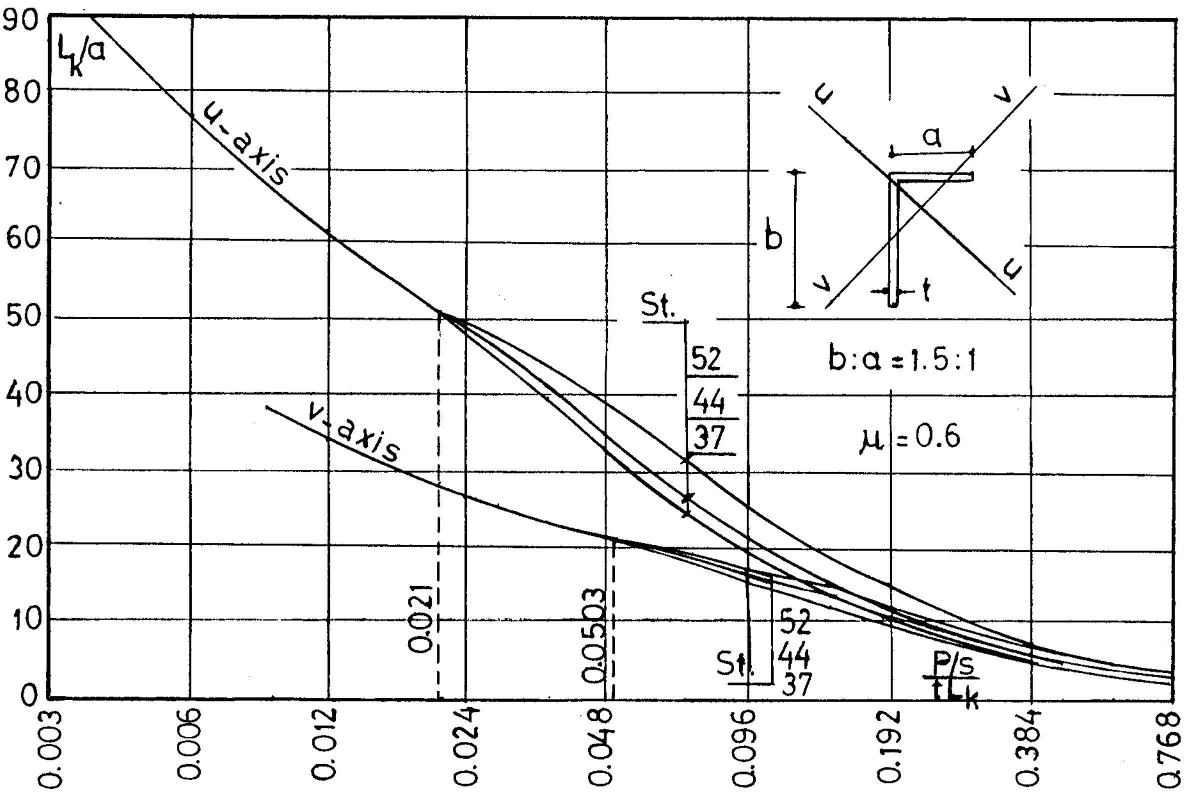


Fig.(1) Optimum dimension of single angle (b/a =1.5).
(Semi-log. scale)

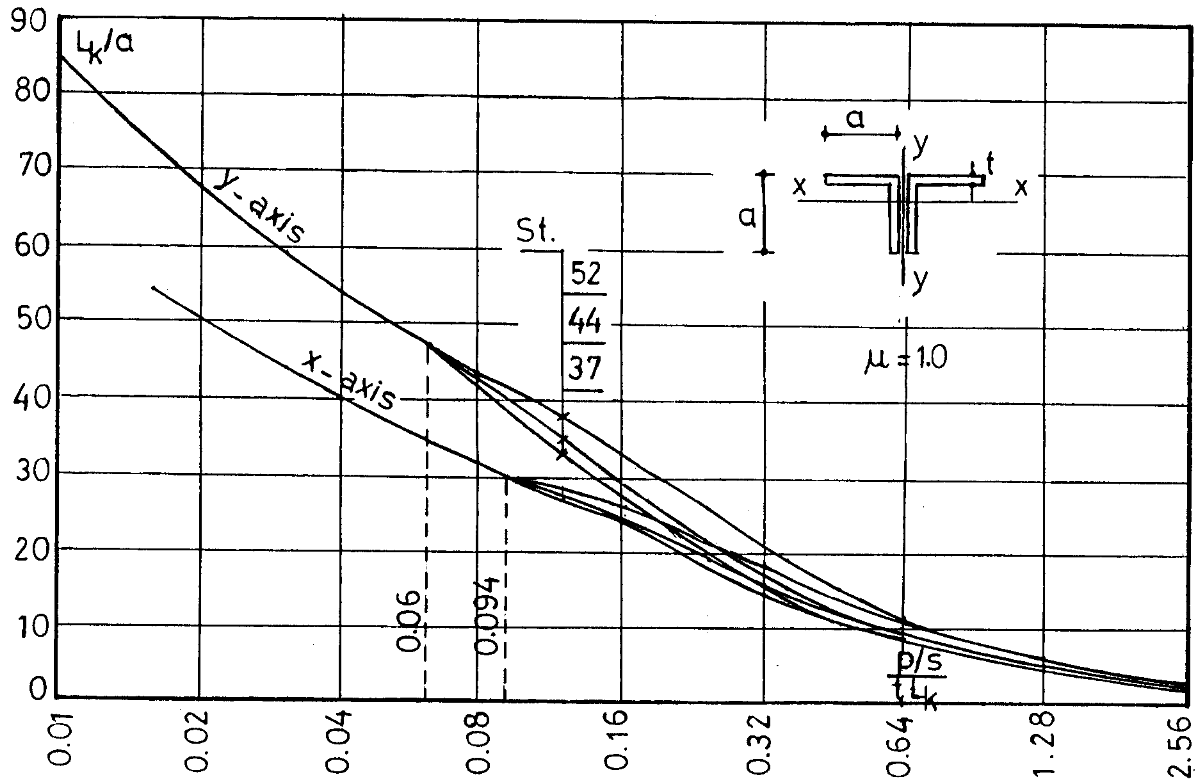


Fig.(2) Optimum dimensions of 2-equal angles (back-to-back).
(Semi-log. scale)

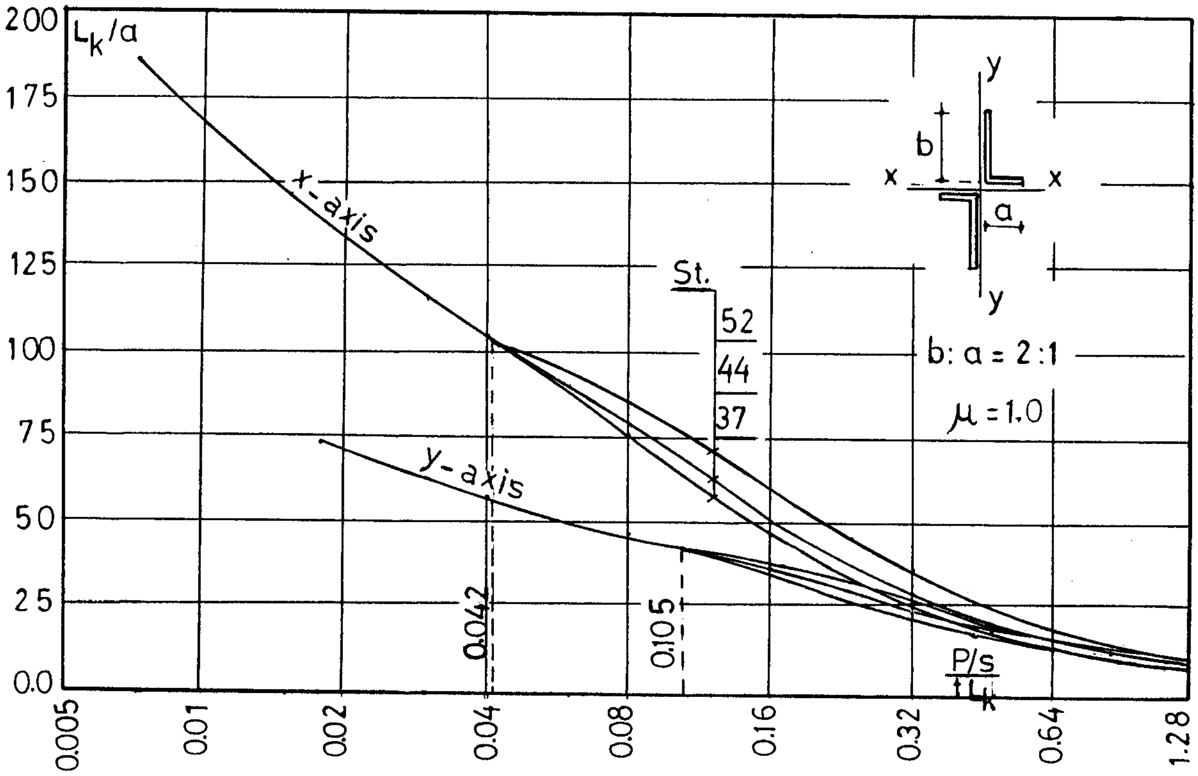


Fig.(3) Optimum dimensions for 2-angles (star-shape) with unequal legs (b/a = 2.0). (semi-log. scale).

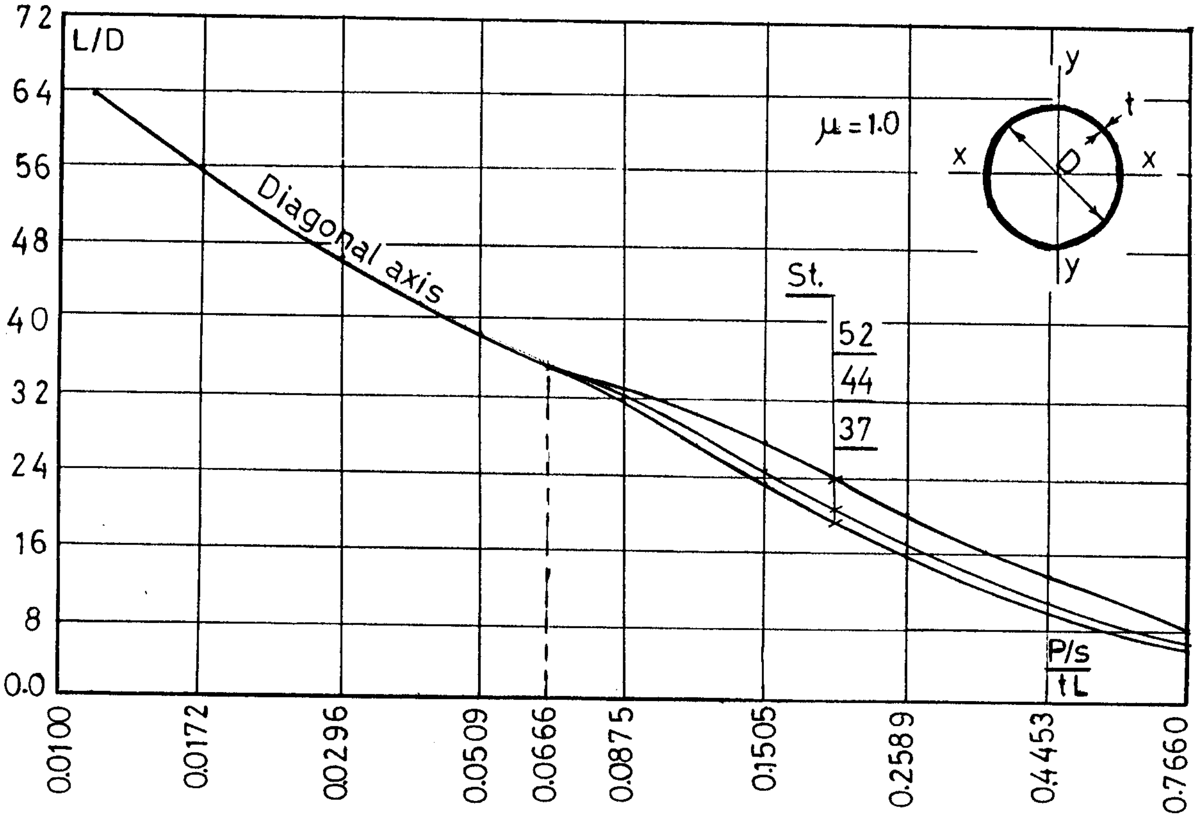


Fig.(4) Optimum dimensions of circular tube section (Semi-log scale).

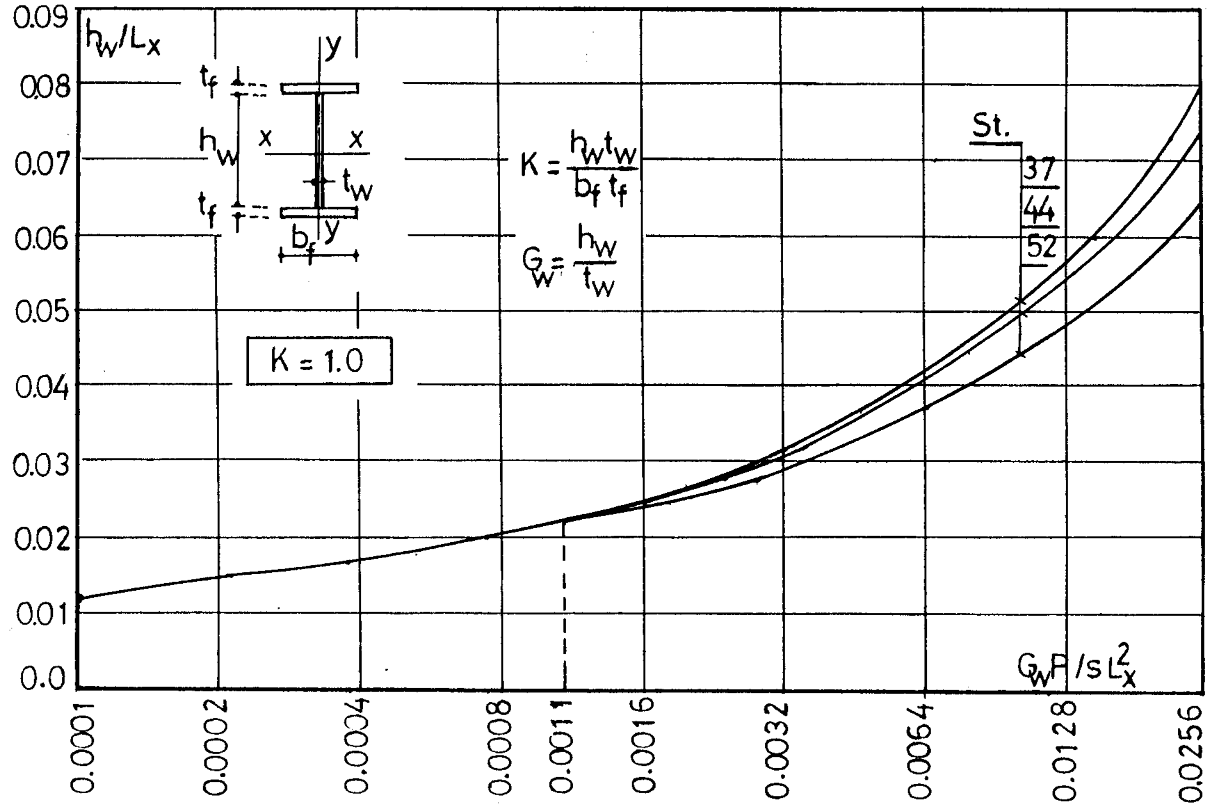


Fig. (5) Optimum dimensions for I-section (buckling about x-axis).
(Semi-log. scale)

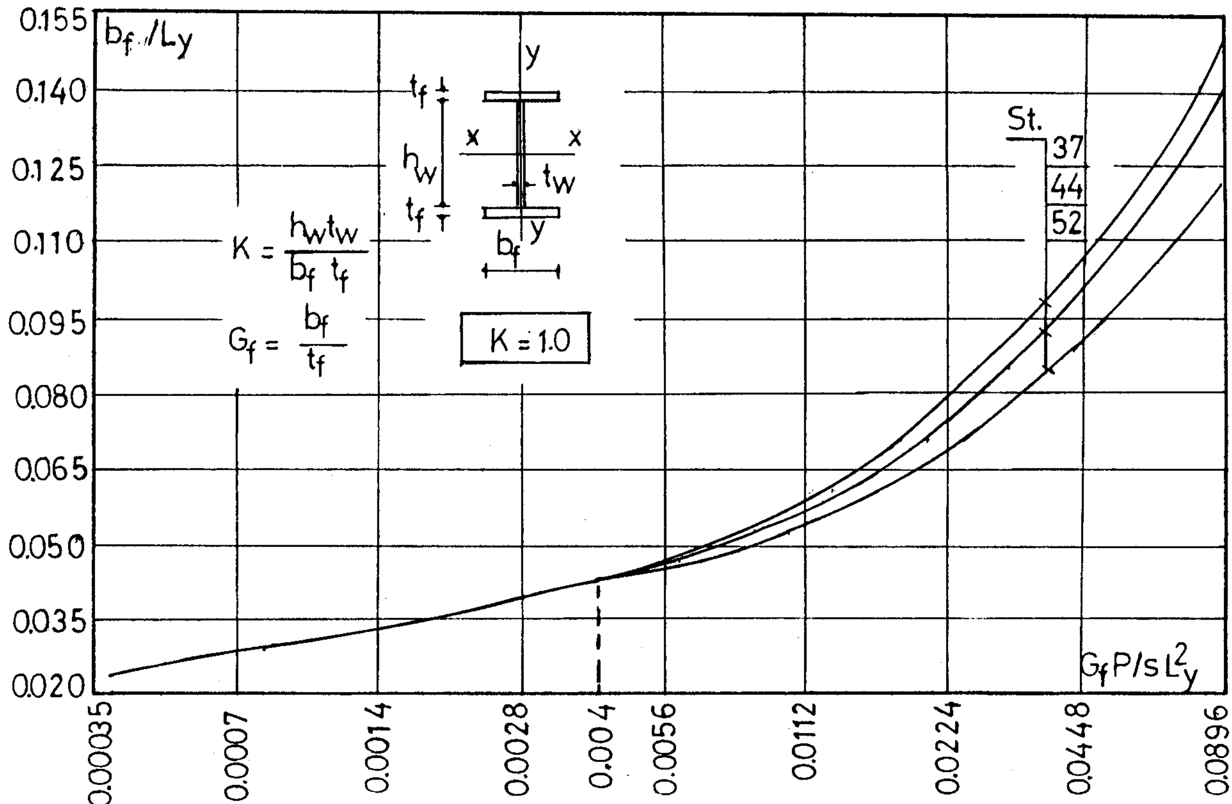


Fig. (6) Optimum dimensions for I-section (buckling about y-axis).
(Semi-log. scale)